MECANICA DE FLUIDOS E HIDRÁULICA II
(FLUID MECHANICS II)

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The following topics are covered:

→ External & internal flow
→ Pressure distributions & forces on the aircraft
→ Numerical simulation
→ Computational fluid dynamics

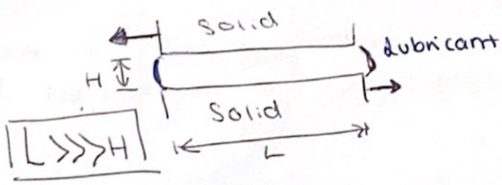
-- Pequeño resumen de qué vamos a dar:

$$u\left(\frac{\partial u}{\partial x}\right) + v\left(\frac{\partial u}{\partial y}\right) = \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

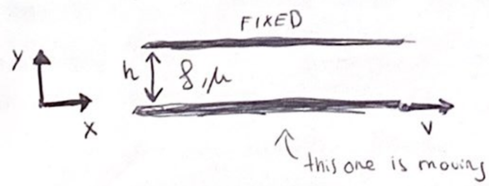
$$Re = \frac{\text{Inertial}}{\text{viscous}} \qquad \boxed{f(\text{viscous}) >> f(\text{inertial})}$$

$$Re <<< 1$$

when this happens it's called Stokes flow, and it could happen in a situation like this:



A point flow is when there is a fluid between two moving plates:



We assume the following:
1. Steady state
2. 1D
3. Newtonian
4. No gravity
5. Incompressible
6. Pression gradient is zero

so we have the continuous equation for incompressible flow:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad ; \quad \text{y que el gradiente de presión es zero}$$

Another type of flow is when there is two fixed planes, called Poiseuille

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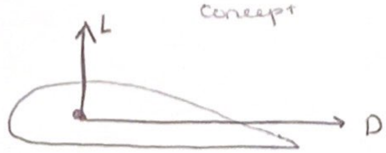
A $\underbrace{\text{potential fluid}}_{\text{concept}}$ for an airfoil has to assume:

- Inviscid } Ideal fluid
- Incompressible
- Irrotational



→ If we want the airfoil to have drag, we can't use the potential fluid concept because it's only for ideal fluids, we need to use: **Bernoulli**

CONTENIDO :

- Fluid - dynamic lubrication
- Intro to the fluids in porous media
- Gas dynamics
- Liquids in ducts
- Laminar & turbulent boundary layer
- Application for the distribution of pressures & forces on the aircraft
- Computational fluid dynamics, practice advanced on fluid dynamics simulators

ASSESMENT :

- Final exam . . . . (min. 5/10)
- Project . . . . (minim. 5/10)
- Homework, lab reports. . . (minimum 5/10)

} First exam period

In order to be evaluated you must have a minimum of 50% attendance

Exercises, tasks	20%
Lab & report	10%
Project	20%
Oral presentation	15%
Final exam	35%

} Second exam period

Recall :

Continuum hypothesis: Materials and transport properties

- Newtonian fluids

Relation Stress vs Rate of strain ; pressure & density variations

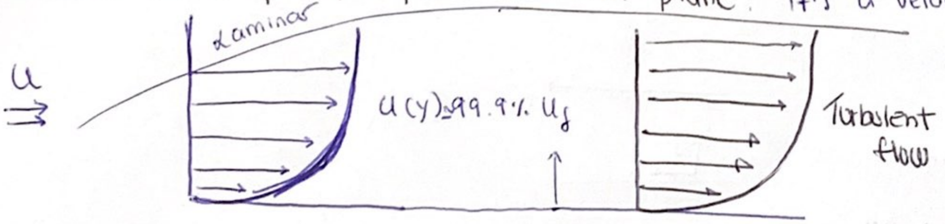
- Reynolds number , Navier - Stokes eqns - additional body forces
interfacial tension : statics , interface deformation , gradients.

Types of effects on fluid :

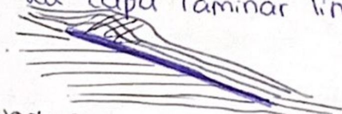
- Von Karma vortex

- Flat plate at zero incidence : the stream lines near the plate are a bit curved due to the viscosity.
we have the boundary layer, the momentum thickness & the displacement thickness.

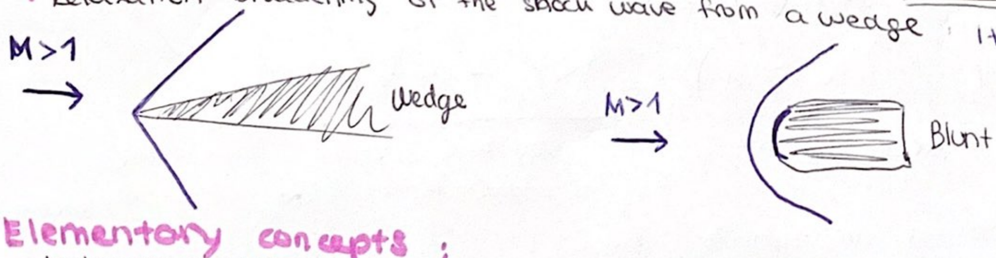
- Blasius boundary - layer profile on a flat plane. it's a velocity profile for a laminar flow



- Leading - edge separation on a plate with laminar reattachment A flat plate 2% thick which is incide 2.5° to the stream. La capa laminar límite se separa en la parte superior en el borde.



- Relaxation broadening of the shock wave from a wedge . It is a shock wave (onda de choque)



Elementary concepts :

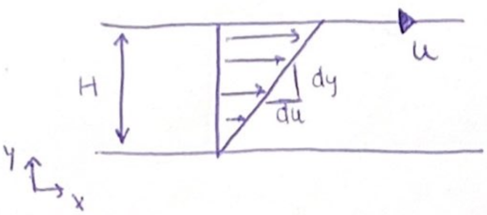
Molecular Dynamics

- Thin films · Experiments on shearing bt 2 molecularly smooth (mica) surfaces separated by thin films of organic liquids. Example: Thin-film photovoltaic cells.

3 types
- Films > 10 molecular diameters can be described in terms of bulk properties
- Thinner films: molecular ordering, quantization of some properties. "effective viscosities" $> 10^5$ bulk value
- Film w/ thickness less than 5 molecular diameters: "solid-like" response.

- Viscosity & Newtonian Fluids

τ ≡ SHEAR STRESS (Force/Area)



$$\boxed{\underset{(Pa)}{\tau} = \mu \frac{U}{H}}$$

unidades $\xrightarrow{}$ $\overset{N \cdot S}{\frac{kg}{m\,s^2} \cdot \frac{S}{m^2} \cdot \frac{\cancel{m}}{S \cdot \cancel{m}}} = N \cdot \frac{\cancel{S}}{m^2} \cdot \frac{\cancel{m}}{\cancel{S} \cdot \cancel{m}} = \frac{N}{m^2} = \underline{\underline{Pa}}$; $N = kg \frac{m}{s^2}$

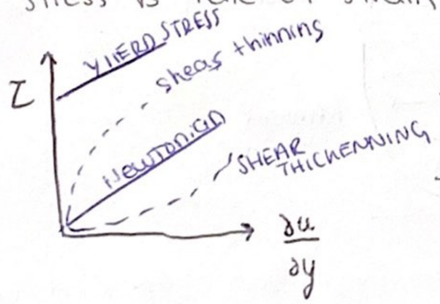
donde μ ≡ shear viscosity

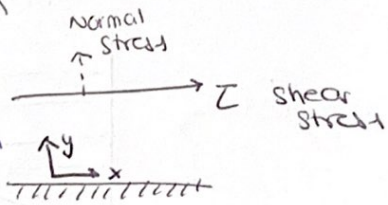
$\tau \underset{\text{Proporcional}}{\propto} \frac{\partial u}{\partial y}$; $\tau\,(\text{shear stress}) = \mu \frac{du}{dy}$ (Newtonian)

Por lo tanto u ≡ velocity $= y \cdot \frac{V}{h}$

- On to equations of motions

(a) stress vs rate of strain





(b) Navier-Stokes equations

Assume that the material properties ρ & μ are constant

MASS BALANCE (continuity) $\nabla \cdot u = 0$

LINEAR MOMENTUM BALANCE $\underbrace{\rho\left(\frac{\partial u}{\partial t} + u \cdot \nabla \cdot u\right)}_{\text{ACCELERATION}} = \underbrace{-\nabla p + \mu \nabla^2 u}_{\text{SURFACE FORCES}} + \underbrace{\rho \cdot g}_{\text{BODY FORCES}}$ ← Navier Stokes

⟹ EACH TERM FORCE/VOLUME

(c) pressure changes accompanying flow

2

along the stream line.

+ on to equations of motions

(d) incompressibility $(\nabla \cdot u = 0)$ $\qquad \nu = \frac{1}{\rho}$

variation of density accompanying motion should be small $(\Delta\rho \ll \rho)$

$$\Delta\rho \approx \frac{\partial \rho}{\partial p} \cdot \Delta p \quad , \quad c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \equiv \quad c = \text{speed of sound}$$

$$\text{Mach} \equiv M = \frac{U}{c}$$

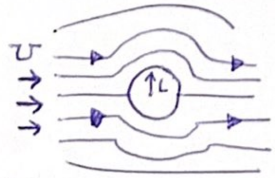
★ Rembember, for ideal gas:

$$c = \sqrt{\gamma R T}$$

- inertially dominated flows : $U/c \ll 1$
- viscously dominated flows : $(U/c)^2 \ll \rho \cdot U \cdot \ell / \mu$

(e) Reynolds number

$$Re = \frac{\rho \cdot U \cdot \ell}{\mu} = \frac{U \cdot \ell}{\nu} \qquad \text{pq} \quad \nu = \frac{1}{\rho}$$



Low-Reynolds-number motions: lubrication, film coating, suspensions, MEMS, ... $\Rightarrow \quad 0 = -\nabla p + \mu \cdot \nabla^2 u$

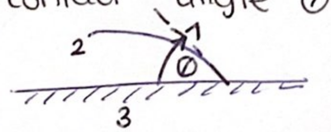
Es la relación entre las fuerzas inerciales y las fuerzas viscosas

- interfacial tension (Force / Lenght or energy / area)

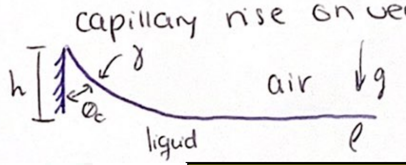
(a) Statics

capillary lenght : $\ell_{cap} \left(\frac{\gamma}{\rho g}\right)^{1/2}$

contact angle $\varnothing_c$ $\qquad \gamma_{12} \cdot \cos \varnothing_c = \gamma_{13} - \gamma_{23}$



capillary rise on vertical planes & fibers

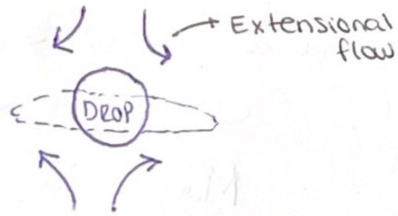


$$h = f(\varnothing_c)\left(\frac{\gamma}{\rho g}\right)^{1/2}$$

Mirar efecto Marangoni

(b) dynamics

- drop deformation, formation of emulsions



DEFORMATION $\propto f\left(\frac{\mu \cdot G a}{\gamma}\right)$

$G \equiv$ shear rate

$a \equiv$ drop radius

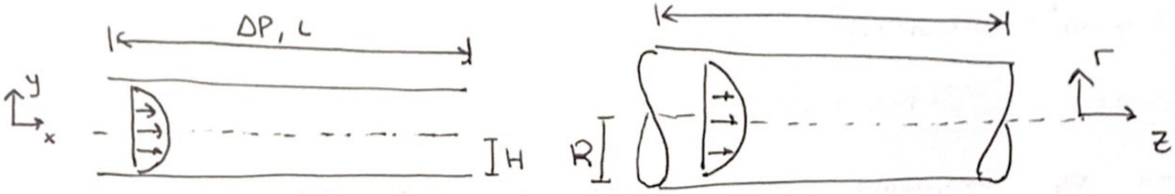
- drop spreading

PROTOTYPICAL FLOWS

Types → ① Steady pressure Driven Flow
→ ② Condition flow

1. Steady pressure Driven Flows

CHANNEL Y PIPES FLOWS



NO SLIP ON BOUNDARIES

$$u(y) = \frac{H^2}{2}\frac{\Delta P}{\mu L}\left[1-\left(\frac{y}{H}\right)^2\right]$$

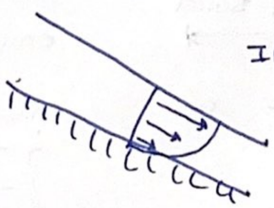
AVG. VELOCITY $\langle u \rangle = \frac{R^2}{8\mu}\cdot\frac{\Delta P}{L}$

$$u(r) = \frac{R^2}{4\mu}\cdot\frac{\Delta P}{L}\left[1-\left(\frac{r}{R}\right)^2\right]$$

MASS FLOW RATE : $Q = \frac{\pi}{8}\frac{\rho}{\mu}\cdot R^4\cdot\frac{\Delta P}{L}$

PARABOLIC (POUSILLE) VELOCITY PROFILE

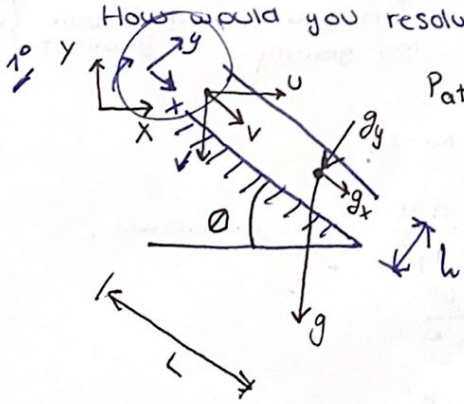
Applications :
- Blood flow
- Pipe flow
- Mems
- Film flows :



Inclined film

How would you resolve this?

1º



Patm

2º Assuming :
1) 1 Dimension
2) Steady Rate
3) Incompressible
4) $\Delta P/\Delta L = 0$

3º we write down the Governing equation:

→ Continuity eq : $\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{0} = 0$

→ Momentum eq (N-S) :

$$\rho\left[\cancel{\frac{\partial u}{\partial t}}^{0} + u\cdot\cancel{\frac{\partial u}{\partial x}}^{0} + v\cdot\cancel{\frac{\partial u}{\partial y}}^{0}\right] = \cancel{\frac{-\partial p}{\partial x}}^{0} + \rho\cdot g_x + \mu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{0} + \frac{\partial^2 u}{\partial y^2}\right)$$

$0 = g\cdot\sin\theta + \mu \quad \partial^2 u$

External flow :

$Re = 5 \cdot 10^5$ laminar

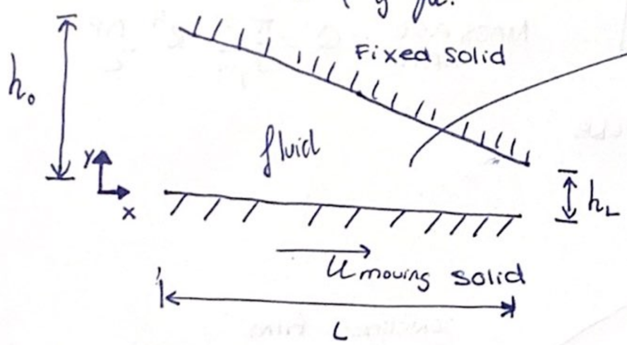
Transitiona

$Re \geq 3.6 \cdot 10^6$ Turbulent

New concept : $\boxed{\text{Lubrication}}$

Invented by Reynolds (1886), pero ya se utilizaba en práctica por los egipcios, ya que echaban agua para que las rocas resbalaran y pudieran moverlas mejor.

Ejemplo : Tenemos 2 solidos, con uno en movimiento de característic-as ρ y μ.

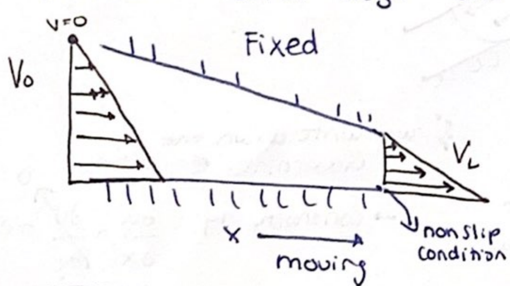


ρ, μ Donde $L >>> h(x)$

Este es un $\boxed{\text{covette flow}}$ porque una pared se está moviendo

Entonces, sabemos que en la pared de abajo aplicaremos non-slip condition y en la de arriba, el fluido tenará $u=0$.

Graficamente será algo así :



Asumimos :

- 1. Dimension
- S.S.
- Incompressible
- No gravity
- No pressure gradient

x - component (N-S)

$$\rho \cdot u \cdot \frac{du}{dx} = \mu \cdot \frac{d^2u}{dy^2}$$ cond. iniciales.

$$\rho \cdot \cancel{U} \cdot \frac{U}{L} = \mu \cdot \frac{\cancel{U}}{h^2}$$

$$\frac{\rho \cdot U}{L} = \mu \cdot h^2$$

$$\frac{\rho \cdot U}{L} = \mu \cdot h^2$$

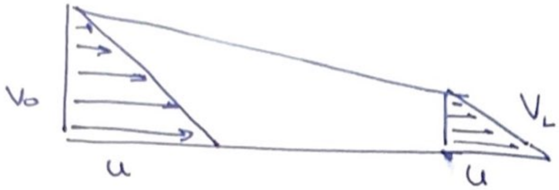
$$\underbrace{\frac{\rho \cdot U}{\mu} \cdot \frac{L}{L}}_{Re} \cdot \frac{1}{L} = \frac{1}{h^2}$$

entonces : $\boxed{\rho ... h^2 ... 1}$ → $\boxed{Re\, h^2 ...}$ → STOKES FLOW

And It satisfies the Stokes ...

¿Que problema hay con el dibujo?

Que no satisface la ec. de continuidad



$$V_0 = \left(u - \frac{u}{h_0} y\right)$$

$$V_L = \left(u - \frac{u}{h_L} \cdot y\right)$$

$$\dot{m}_0 = \dot{m}_L = \rho_0 \cdot V_{Avg_0} \cdot A_0 = \rho_L \cdot V_{Avg_L} \cdot A_L$$

Ec. continuidad

Hallamos Avg. velocity:

- $V_{Avg_0} = 1/A_{cs} \int_A V_0 \cdot dA = \dfrac{1}{h_0 \times 1} \displaystyle\int_0^{h_0} \left(u - \frac{u y}{h_0}\right) dy$

$$\cancel{V_{Avg_L} = 1/h_L} = \frac{1}{h_0}\left[\left(uy - \frac{uy^2}{h_0 2}\right)\right]_0^{h_0} = \frac{1}{\cancel{h_0}}\left(u\cancel{h_0} - \frac{u h_0^{\cancel{2}}}{h_0 \cancel{2}}\right)$$

$$\left[V_{Avg_0} = \frac{u}{2}\right]$$

- $V_{Avg_L} = \dfrac{1}{h_L} \displaystyle\int_0^{h_L} \left(u - \frac{uy}{h_L}\right) dy = \frac{u}{2}$

- $\cancel{\rho_0} \dfrac{\cancel{u}}{2} \cdot h_0 = \cancel{\rho_L} \cdot \dfrac{\cancel{u}}{2} \cdot h_L$

$$h_0 \neq h_L$$

x - component (NS)

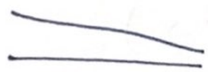
$$\rho \cdot \frac{\partial u}{\partial t} + \cancel{u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y}} = -\frac{\partial P}{\partial x} + \rho g_x + \mu\left(\frac{\partial u^2}{\partial x^2} + \frac{\partial u^2}{\partial y^2}\right)$$

$$\boxed{\frac{dP}{dx} = \mu \frac{d^2 u}{dy^2} = cte}$$

Boundary cond:
$y = 0 \quad u = u$

$$\left[\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x}\cdot h^3(x)\right] = 6\cdot\mu\cdot U\cdot\frac{dh(x)}{dx}\right]$$

$$P(0) = P_\infty = P(L)$$



$$\left[h(x) = h_0 + (h_0 - h_L)\cdot\frac{x}{L}\right]$$

Se acaba la clase, pero la solución es:

$$\left[\frac{P(x) - P_\infty}{\mu\cdot\frac{U\cdot L}{h_0^2}} = \frac{6\left(x/L\right)\left(1-\frac{x}{L}\right)\left(1-\frac{h_L}{h_0}\right)}{\left(1+\frac{h_L}{h_0}\right)\left(1-\left(1-\frac{h_L}{h_0}\right)\frac{x}{L}\right)^2}\right]$$

LUBRICATION

* Velocity and pressure profile

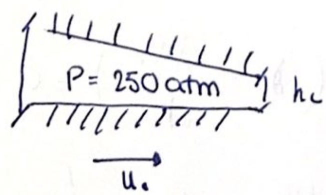
Example

SAE = 50 $\qquad u = 10\ m/s \qquad L = 4\ cm \qquad h_0 = 0.1\ mm$

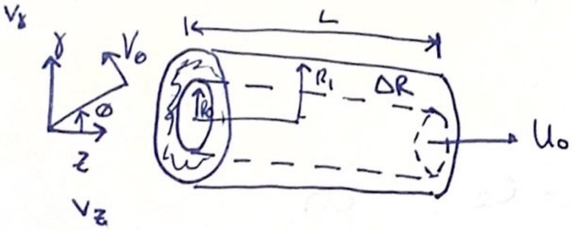
Resolución:

Como ya conocemos h_0, primero hallamos:

$$\mu \frac{UL}{h_0^2} = (2.5 \cdot 10^7\ Pa) = 250\ atm$$



Ahora lo resolvemos para un cilindro 3D:



Suponemos que el cilindro interior se mueve con velocidad U_0, y el fluido está entre el interior y el exterior.

$V : (V_r\ ;\ V_\theta\ ;\ V_z)$

$V : (0\ ;\ 0\ ;\ V_z)$

$\Delta R = R_1 - R_0 <<< L$

Asumimos:

1) Steady state
2) 1D in z-direction
3) Incompressible
4) No gravity
5) Pressure & temperature gradient are neglected

Governing equation:

· Continuity:

$$\frac{1}{\gamma}\underbrace{\frac{\partial(\cancel{r V_r})}{\partial r}}_{0} + \frac{1}{r}\underbrace{\frac{\partial \cancel{V_\theta}}{\partial \theta}}_{0} + \frac{dV_z}{\partial z} = 0\ ;$$

$$\frac{\partial V_z}{\partial z} = 0 \longrightarrow V_z(r) = f(r)$$

· Momentum (NS)

z-component (Navier stokes)

$$\rho\left(\underbrace{\cancel{\frac{\partial V_z}{\partial t}}}_{0\ \text{steady state}} + V_r \cdot \underbrace{\cancel{\frac{\partial V_z}{\partial r}}}_{0} + \underbrace{\cancel{\frac{V_\theta}{r} \cdot \frac{\partial V_z}{\partial \theta}}}_{0} + V_z \cdot \cancel{\frac{\partial V_z}{\partial z}}\right) = \underbrace{\cancel{-\frac{\partial p}{\partial z}}}_{\text{no hay gradiente} \to 0} + \underbrace{\cancel{\rho} \cdot g_z}_{\text{no hay gravedad} \to 0} + \mu\left[\frac{1}{\gamma} \cdot \frac{\partial (r \cdot \partial V_z / \partial r)}{\partial r}\right] +$$

$$+ \mu\left[\frac{1}{r^2} \cdot \frac{\partial^2 V_z}{\ldots} + \frac{\partial^2 V_z}{\ldots}\right]$$

$\gamma\ \left[\frac{\partial r}{\ldots} \frac{\partial z}{\ldots}\right]$

Boundary conditions

$r(R_0) = U_0$

$r(R_1) = 0$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(\frac{r \cdot \partial V_r}{\partial r}\right)\right] = 0$$

Solving the diferential eq by separation of variables

$$\frac{d}{dr}\left(r \cdot \frac{dV_r}{\partial r}\right) = 0$$

$$r \cdot \frac{dV_r}{\partial r} = C_1 \qquad V_r = C_1 \cdot Ln(r) + C_2$$

$$\frac{dV_r}{dr} = \frac{C_1}{r}$$

$V_z(R_0) = U_0 = C_1 \cdot Ln(R_0) + C_2$ (1)

$V_z(R_1) = 0 = C_1 \cdot Ln(R_1) + C_2$ (2)

$C_1 - C_2 \Rightarrow \quad U_0 - 0 = C_1 \cdot Ln(R_0) + \cancel{C_2} - [C_1 \cdot Ln(R_1) + \cancel{C_2}]$

$$U_0 - 0 = C_1 [\ln(R_0) - Ln(R_1)]$$

$$\left[C_1 = \frac{U_0}{Ln(R_0/R_1)} \right]$$

$$C_2 = -C_1 \cdot Ln(R_1)$$ Reeplazo C1 en ecuación 2.

$$V_z(r) = \frac{U_0}{Ln(R_0/R_1)} [\ln(r) - Ln(R_1)]$$

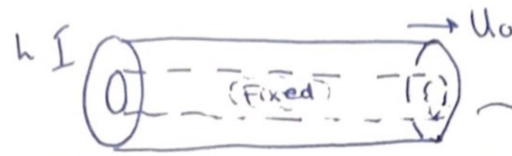
$$\boxed{V_z(r) = \frac{U_0 \cdot Ln(r/R_1)}{Ln(R_0/R_1)}}$$

Shear stress

Inner

★ + velocity & pressure profile

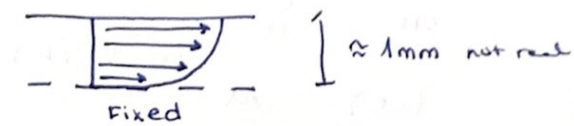
Example 2



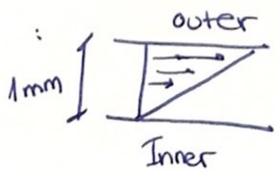
El cilindro exterior se mueve y el interior está quieto, al contrario que en el ejemplo 1.
Así que ya sabemos V_z :

$$\boxed{V_z = \frac{U_0 \cdot \ln(r/R_0)}{\ln(R_1/R_0)}}$$

Si hacemos zoom en el fluido



Si el hueco h es muy pequeño o más grande
1 mm :



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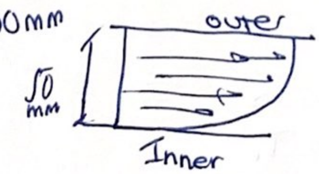
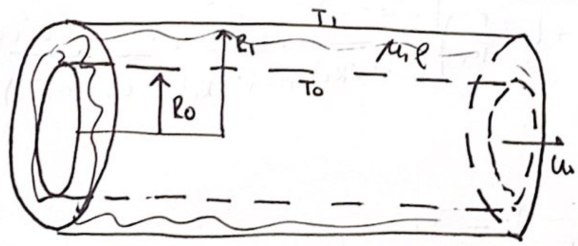


★ Heat transfer between cocentric cylinders (same example as before)



El fluido se encuentra entre ambos cilindros, el interior se mueve.
Ya conocemos :

$$V_z = \frac{U_0 \ln(r/R_1)}{\ln(R_0/R_1)}$$

Necesitamos the Energy equation :

$$\rho \cdot C_p \Big(\underbrace{\cancel{\frac{\partial T}{\partial t}}^{\,0} + \cancel{V_r \frac{\partial T}{\partial r}}^{\,0} + \cancel{\frac{V_\theta}{r} \frac{\partial T}{\partial \theta}}^{\,0} + \overset{\neq 0}{V_z \cdot \frac{\partial T}{\partial z}}}_{\text{convection}} \Big) = -\overset{cte}{K} \Big[\overbrace{\frac{1}{r} \cdot \frac{\partial\left(r \cdot \frac{\partial T}{\partial r}\right)}{\partial r} + \cancel{\frac{1}{r^2} \cdot \frac{\partial^2 T}{\partial \theta^2}}^{\,0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{\,0}}^{\text{conduction}} \Big] + \Phi$$

$\to 0$

$\Phi \to$ Due to friction: VISCOUS DISSIPATION

La dirección de heat transfer is predomently ↗ in the r direction, así que $V_z \cdot \frac{dT}{dz} \sim 0$, en otras
y $T(r)$.

Viscous dissipation : $\Phi = \mu \Big[2\Big(\Big(\frac{\partial \cancel{V_r}}{\partial r}\Big)^2 + \Big(\frac{1}{r} \cdot \frac{\partial V_\theta}{\partial \theta} + \frac{\cancel{V_r}}{r}\Big)^2 + \Big(\frac{\partial V_z}{\partial z}\Big)^2 \Big) + \Big(\frac{1}{r} \cdot \frac{\partial \cancel{V_z}}{} + \frac{\partial \cancel{V_\theta}}{}\Big)^2 +$

. . .

Separation of variable

$T(r)$

$$r \cdot \frac{dT}{dr} = \frac{\mu}{k} \cdot \frac{U_0^2}{Ln^2(R_0/R_1)} \cdot \ln r + C_1$$

$$\frac{dT}{dr} = \frac{\mu}{k} \cdot \frac{U_0^2}{Ln^2(R_0/R_1)} \cdot \frac{Ln\, r}{r} + \frac{C_1}{r}$$

$$T(r) = \frac{\mu}{k} \cdot \frac{U_0^2}{Ln^2(R_0/R_1)} \cdot \frac{Ln^2 r}{2} + C_1 \cdot \ln r + C_2$$

BC'S

$T(R_0) = T_0$

$T(R_1) = T_1$

$$T(r) = \frac{\mu \cdot U_0^2}{2k \cdot Ln^2(R_0/R_1)} \cdot \left[\ln^2(r) - Ln^2(R_1) \right] + Ln\left(\frac{r}{R_1}\right) \left[\frac{(T_0 - T_1)}{Ln(R_0/R_1)} - \frac{\mu U_0^2}{Ln^2(R_0/R_1)} \cdot \frac{Ln^2(R_0) - Ln^2(R_1)}{Ln(R_0/R_1)} \right] + T_1$$

Only by conduction

$$\boxed{T(r) = \frac{T_0 - T_1}{Ln(R_0/R_1)} \cdot Ln\left(\frac{r}{R_1}\right) + T_1}$$

by conduction:

$$T(R_0) = T_0 = T_0 - \cancel{T_1} \cdot \frac{\cancel{Ln(R_0/R_1)}}{\cancel{Ln(R_0/R_1)}} + \cancel{T_1} \checkmark$$

$$T(R_1) = T_1 = (T_0 - T_1) \frac{\cancel{Ln(R_1/R_1)} \to 0}{Ln(R_0/R_1)} + T_1 = T_1 \checkmark$$

Heat transfer on the creeping

ENERGY EQUATION

$$\rho C_p \left(\frac{\partial T}{dt} + \varphi \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K \nabla^2 T + \Phi$$

Assumptions

1) Steady State flow
2) Temperature gradient is on x-axis
3) 1D flow u-component
4) $\Phi = 0$

$$\left[\rho C_p u \frac{\partial T}{\partial x} = K \nabla^2 T \right]$$

Dimensionless Analysis

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial v^*}{dy^*} = \frac{1}{Re} \nabla^{*2} u^*$$

$Re >>> 1$

$Re <<< 1$

$$X^* = \frac{x}{L} \qquad u^* = \frac{u}{U}$$

$$T^* = \frac{T}{T_1 - T_0}$$

$$\left[u = \frac{\partial T}{\partial x} = \frac{K}{\rho C_p} \nabla^2 T \right]$$

$$u^* U \frac{\partial T^*}{\partial x^* L} (T_1 - T_0) = \frac{K}{\rho C_p} \frac{\nabla^{*2} T^* (T_1 - T_0)}{L^2}$$

$$\boxed{u^* \frac{\partial T^*}{\partial x^*} = \frac{K \cdot \cancel{L}}{\rho \cdot C_p \cdot L^{\cancel{2}} U} \nabla^{*2} T^*}$$

adimensional

$$\frac{K}{\rho C_p \cdot U \cdot L} = \frac{1}{Pe}$$

Peclet Number

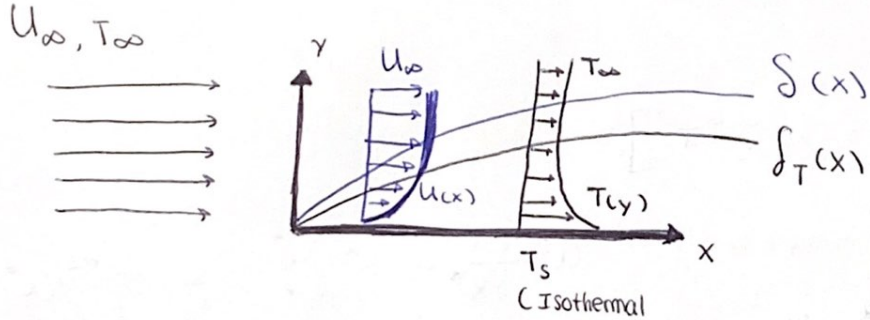
* Juguemos con Peclet number:

If the Prandtl number:

$Pr \downarrow\downarrow\downarrow$ (low) $\Longrightarrow$ Conductive Transfer is Strong $\uparrow\uparrow\uparrow$
(Liquid metals)

$Pr \uparrow\uparrow\uparrow$ (high) $\Longrightarrow$ Convective Transfer is strong $\uparrow\uparrow\uparrow$
(water, oil)

Thermal Boundary Layer



$$\frac{\delta}{\delta_T} = Pr^n$$

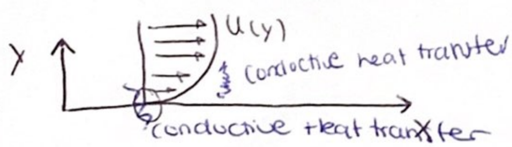
(n = positive number

$$\frac{T(y) - T_\infty}{T_s - T_\infty} = 0.99 \Rightarrow \delta_T$$

For oil, water
$\delta >> \delta_T$

For liquid metals
$\delta << \delta_T$

For gases
$\delta \cong \delta_T$



$u(0) = 0$ no-slip condition

$$q_s'' = -k \left.\frac{\partial T}{\partial y}\right|_{y=0} \quad \text{Fourier's law}$$

$$q_s'' = h\,(T_s - T_\infty) \quad \text{Newton's cooling law}$$

$$h \text{ (heat thermal conductivity)} = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{(T_s - T_\infty)}$$

Experimental results for the local heat transfer coefficient for flow over a flat surface

$$h_x(x) = a\,x^{-0.1}$$

where a is a coefficient $\left(\frac{W}{m^{1.9}-K}\right)$ and x (m) distance from the L.E.

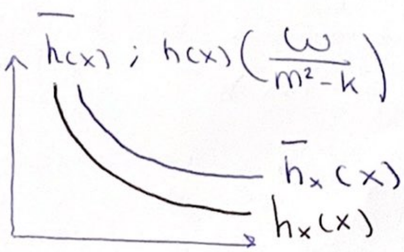
Determine the ratio of the avergge $\overline{h}_x(x)$ to the local h_x heat transfer coefficient.

Solution:

$$\overline{h_x}(x) = \frac{1}{X}\int_0^x h_x(x)\,dx$$

$$\overline{h_x}(x) = \frac{1}{x}\int_0^x a\,x^{-0.1}\,dx \; ; \; \overline{h}_x(x) = \frac{a}{x}\left(\frac{x^{0.9}}{0.9}\right) = 1.11\,a\,x^{-0.1}$$

$$\frac{\overline{h_x}(x)}{h_x(x)} = 1.11$$



Enunciado:

* $U = 10\,m/s$; the chord lenght is $1\,m$ and the flow (a) air and (b) water, at sea level conditions.

Determine thermal boundary layer

Answer:

$$Re = \frac{\rho \cdot U L}{\mu} = \frac{1{,}21 \times 10 \times 1}{1{,}789 \cdot 10^{-5}} = 676355{,}51$$

In case air (a)

Asumimos que es laminar flow

* The Prandtl # for air is $Pr \simeq 0.7$

$$\frac{\delta}{\delta_T} = Pr^{1/3} \; ; \; \delta = \frac{5 \times X}{\sqrt{Re}} = \frac{5 \times 1}{\sqrt{676355}} = 6.07 \cdot 10^{-3}\ m$$

Pr

(b) for water

$$Re = \frac{1000 \cdot 10 \cdot 1}{0.001} = \frac{\rho \cdot U \cdot L}{\mu} = 1.0 \cdot 10^{7} \rightarrow \text{sería turbulento}$$

entonces $\delta = \dfrac{0.38 \times X}{(Re)^{1/5}} = 0.0151\ m$; $\delta_T = \dfrac{\delta}{(P_r)^{1/3}} = 8.05 \cdot 10^{-3}\ m$

it's very low, so heat transfer will be

* For water $P_r \simeq 6,6$

Remember! You can calculate the Prandtl number, if you need to:

$$\boxed{P_r = \frac{C_p \cdot \mu}{K}}$$

NUSSELT NUMBER

$$\overline{N_{um}} = \frac{\overline{q_w} \cdot L}{K (T_w - T_\infty)} = \frac{\overline{h} \cdot L}{K} = \frac{\text{convection}}{\text{conduction}}$$

$$q'' = h\,(T_s - T_\infty)$$ convection heat transfer

$$h = \frac{q}{(T_s - T_\infty)}$$

- caso (esfera). Longuitud característica (L =

  high order terms

  $$N_{um} = 2.0 + 0.5\ P_r \cdot Re + O(P_r^2 \cdot Re^2) + \dots\dots$$

- caso (circular cylinder). Longitud característica : (L = 2R)

  $$N_{um} = B - P_r^2 \cdot Re^2 (16 + B^2)$$

  where $B = \dfrac{2}{Ln\left(\frac{8}{P_r \cdot Re}\right) - \gamma}$ donde γ = gamma = 0.577

otra forma de aproximarse

$$N_{um} = 0.42\ P_r^{0.2} + 0.57 \cdot P_r^{1/3} \cdot R_e^{1/2}$$ cuando $0.1 < Re_e < 10^4$

Temperature distribution

(diferentes casos)
para resolver con cada boundary conditi

(1)

Plate T_1

h μ, ρ

Plate T_0

(2)

$k \cdot \frac{\partial T}{\partial y}$

(T_i)

$k \cdot \frac{\partial T}{\partial y} \simeq 0$

Temperatura del fluido cerca de los bordes

↳ when this is equal to 0, the process es adiabatico

(3)

$T_{\infty,1}$ $h_1(T - T_{\infty_1})$

This is convection

$T(y)$

$T_{\infty,0}$ $h_0(T - T_{\infty_0})$

(4) one plate

T_1

fluid 2 μ_2, ρ_2

fluid 1 → μ_1, ρ_1

T_0

cuando 2 fluidos están en contacto, $T_{\delta_1} = T_{\delta_2}$ donde se tocan, no hay gradiente de temperatura.

EJERCICIO

Lubricating oil at 20°C with $\rho = 890 \, kg/m^3$; $\mu = 0.8$ Pa·S
$k = 0.15 \frac{W}{m\text{-}k}$ y $C_p = 1800 \frac{J}{kg\text{-}k}$ is to be cooled by flowing at an average velocity of 2m/s through a 3cm diameter of a circular cylinder the walls are at 10°C
Estimate the heat los $\left(\frac{W}{m^2}\right)$ at $x = 10$ cm

$$\overline{Nu}_m = \frac{\bar{q}_w \cdot L}{K_f (T_w - T_\infty)}$$

$$\overline{Nu}_m = 0.42 \, Pr^{0.2} + 0.57 \cdot Pr^{1/3} \cdot Re_D^{1/2}$$

donde $Re_D = \frac{\rho \cdot u \cdot D}{\mu} = \frac{890 \frac{kg}{m^3} \times 2 \frac{m}{s} \times 0.03 m}{0.8 \, Pa \cdot s} = 66.75$

$0.1 < Re_D < 10^4$

$$P_r = \frac{Cp \cdot \mu}{K} = \frac{(1800 \, J/kg \cdot \cancel{K}) \times (0.8 \, Pa \cdot s)}{0.15 \frac{W}{m \cdot \cancel{K}}}$$

Resultado :

$\overline{Nu}_m = 101.5637$

$$\bar{q}_w = \frac{\overline{Nu}_m \times K (T_w - T_\infty)}{L} = \frac{(101,5637) \times 0.15 \frac{W}{m\text{-}K} \times (283 - 293) K}{0.1 \, m}$$

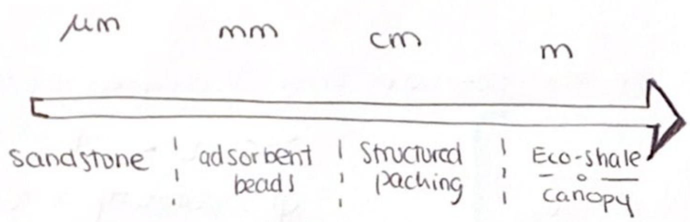
$\boxed{\bar{q} = 15.23 \ldots}$

DYNAMIC FLUIDS IN POROUS MEDIA

POROUS MEDIA: it can be defined as anything that is composed of a solid matrix & voids or simpler a material that contains pores.

It is a body composed of a persistent solid part, called solid matrix, and the remaining void space (or pore space) that can be filled with one or more fluids (e.g.: water, oil, gas...)

- Applications: oil exploration, ground-water flow, heat pipes, filtration, composites processing, wicking & bio heat & Mass transfer.
- Unidades:



Phase is defined in a chemically homogeneous portion of a system under consideration that is separated from other such portions by a definite physical boundary.

→ Single phase: the void space of the porous medium is filled by a single fluid.

→ Multiphase system: the void is filled by 2 or + fluids that are immiscible w/ each other (they maintain a distinct boundary between them).

Void spaces: **TYPES**

A) bt the particles that comprise the matrix (intergranular)

B) within the particles themselves

a a solid matrix & voids.

In order to derive mathematical models for fluid flows in porous media some restrictions are placed upon the geometry of the porous medium:

- (P1) The void space of the porous medium is interconnected
- (P2) The dimensions of the void space must be large compared to the mean free path length of the fluid molecules. ← hypotetical continuum
- (P3) The dimensions of the void space must be small enough so that the fluid flow is controlled by adhesive forces at fluid-solid interfaces & cohesive forces at fluid-fluid interfaces (multiphase systems)
  ↳ excludes cases like a network of pipes from the definition of a porous medium.

POROSITY $n [L^3 L^{-3}]$

Is the volumetric fraction of the medium that is occupied by the voids:

$$\left[\phi = n = \frac{V_v}{V_T} = \frac{V_v}{V_s + V_v} \right] \qquad 0 < n < 1$$

ϕ = porosity $\equiv n$ (misma cosa)

where V_v, V_s, & V_T are the volumes of the voids, matrix solids, and total medium, respectively.

Porosity also defines the avg. fraction of a cross section through a medium that is occupied by boids.
The fraction of the area occupied by voids is = to the porosity.

En $[m^3 m^{-3}]$ para:

- gravel (0.25 - 0.4) ; • sand (0.25 - 0.50)
- silt (0.35 - 0.5) ; • clay (0.40 - 0.70)

- Primary porosity : occurs between the solid particles
- Secondary porosity : caused by fracturing & dissolution.

EFFECTIVE POROSITY $n_e [L^3 L^{-3}]$

Vs

Tortuosity t $[L\ L^{-1}]$

Is the ratio of the avg. travel path to the distance of separation.

The ↑↑↑ t the more complicated path, ⇒ less well-connected pore space

Specific surface area $[L^2\ L^{-3}]$

Describes the area of the interface bt the matrix solids & the void space per unit volume of porous medium.

↑↑↑ Specific surface area   ↑↑ Roughness of the surface of the particles   ↓↓ Size of particles.

Water saturation S_w

The fraction of pore spaces that are filled w/ water & it is defined as the ratio of the volumetric water content to the porosity

$$S_w = \frac{\theta}{n}$$

n → porosity

Gravimetric water content θ_g $[M\ M^{-1}]$

of a medium is = to the ratio of the mass of water in a sample M_w to the mass of oven-dried soil, M_s:

$$\theta_g = \frac{M_w}{M_s}$$

Bulk density ρ_b $[M\ L^{-3}]$

It's equal to the ratio of the total mass of a sample of its total volume.

If a medium only has water & aire in the pores, ρ_b is = to:

$$\rho_b = \frac{M_T}{V_T} = (1-n)\rho_s + \theta\rho_w + (n-\theta)\rho_a$$

Three fluid properties are of primary importance in the study of subsurface hydrogeology.

- **Fluid density** $\rho_f\ [M L^{-3}]$ (ρ_w for water density)

  Is the ratio of the mass of a sample of the fluid to the sample volume

- **Dynamic viscosity** $\mu\ [M L^{-1} T^{-1}]$

  describes the resistance to flow presented by a flowing fluid

$$\mu = \frac{\sigma}{dv/dy}$$

donde
- σ = shear stress applied to the fluid
- y = distance perpendicular to solid surface
- v = velocity parallel to surface.

- **Fluid compressibility** $\beta\ [L^2 M^{-1} L^{-1} T^2]$

  Describes the resistance of the fluid to changing its volume in response to a change in the applied pressure per unit volume of fluid

$$\beta = -\frac{1}{V_0} \cdot \frac{dV}{dP}$$

- V_0 = sample volume before compression
- Fluido incompresible : $\beta = 0$

$$PV = RT \quad ; \quad V = \frac{RT}{P} \quad ; \quad \frac{P}{RT} = \frac{1}{V_0}$$

$$\beta \ -\frac{1}{V_0} \cdot \frac{dv}{dP} = -\frac{\cancel{R}T}{P^2} - \frac{P}{\cancel{RT}} \quad ; \quad \beta = \frac{1}{P}$$

* Permeability $k\ [L^2]$

Describes the ability of the medium to transmit a fluid under an applied potential gradient.

Fluid going through a porous medium can be visualized most simply as a fluid flowing through a collection of interconnected tubes.

At the walls of each tube, fluid velocity = 0.

[↑ velocity ↑ further from tube walls bc of "drag" of walls exerts less influence on fluid.]

$$k = \frac{\ }{(1-n)^2} \cdot \frac{\ }{180}$$

d_m = ... particle size distribution